Development of a Local Instructional Theory for the Sequences and Series Concept Based on Contextual Teaching and Learning

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Abstract---The result of the preliminary research study indicated that the learning trajectory of sequences and series in school textbooks was not able to develop students’ problem-solving ability, the mathematics belief that students were not fostered well. Therefore, it is necessary to design a learning trajectory based on students’ experiences. This research was conducted to produce Local Instructional Theory (LIT) for the sequences and series concept based on Contextual Teaching and Learning (CTL). Theoretical development is driven by an iterative process of designing instructional activities, performing teaching experiments, and conducting retrospective analysis to contribute to local instruction theory on the concept of sequences and series. The subject of this research is students of senior high school in Indonesia. The interview, observation, and distribution of the questionnaire, together with the test were done to get data for this research. The data analysis technique used is descriptive analysis and statistical analysis. Form the research result show that LIT can be used for all students, and help them find concepts and develop students’ thinking ability for problem-solving.

Keywords---contextual teaching, learning, local instructional, problem-solving skill, sequences, series concept.
Introduction

Sequences and series are the topics that students learn in a mathematics class in high school that aim to develop their ability in arithmetic or geometric sequence patterns to solve contextual problems. The ability of students to understand the material of sequences and series is very important to be developed so that they can solve problems properly. However, the facts indicate the students’ problem-solving abilities are still low, especially in solving real-world problems (Ekowati, 2015; Mukwambo, 2016; Pinwanna, 2015). The majority of students still have difficulty in understanding the sequence and series material, especially in problem-solving tests (Gee, 2019; Indriani et al., 2018).

Furthermore, the mechanistic learning method and the straight teaching delivery pattern of teachers at the formal level of mathematics are considered as the problem faced by the students. For example, without providing any stimulus at the starting of the class by telling the topic relation and application to daily life, the teacher directly conveys the material. The teacher only gives series of numbers and determines the formula or pattern of the numbers (Fauzan et al., 2020; Gee, 2019; Indriani et al., 2018; Tohir & Abidin, 2018). The teacher does not provide and guide the students to find mathematical concepts and solve problems in their way, but tends to focus students on remembering formulas so that when it comes to a given question with a slightly high level of ability or a question that is different from the example, the students are unable to answer it correctly (Gee, 2019; Tohir & Abidin, 2018). It is the essential for students to be involved in terms of finding concepts and applying them to solve life problems in learning mathematics. For this reason, it is necessary to have a learning design that can improve the quality of learning and is also able to train students’ abilities in solving mathematical problems through the design of mathematics learning paths. The learning flow contains hypotheses (allegations) regarding student learning activities following the learning activities (plots) designed by the teacher in the form of a Hypothetical Learning Trajectory (HLT) (Bustang et al., 2013; Hendrik et al., 2020; Meika et al., 2019; Simon et al., 2018).

In mathematics learning, HLT provides an overview of the learning activities or tasks given to students to elicit the expected response (hypothesized) to achieve learning objectives (Nuraida & Amam, 2019; Prahmana & Kusumah, 2016). The HLT that has been tested will be in the form of LIT (Larsen, 2013; Nickerson & Whitacre, 2010). LIT is in the form of a theory about how to teach a certain material (Larsen, 2013; Nickerson & Whitacre, 2010). LIT consists of theories about how to teach certain topics and theories about things that support learning (Gravemeijer, 2015; Prahmana & Suwasti, 2014; Risdiyanti & Prahmana, 2018). Furthermore, LIT supports teachers in creating the right learning environment for students to understand certain concepts and relate them to daily activities (Bustang et al., 2013; Ulfa & Wijaya, 2019).

Providing opportunities for students to further explore the uses and benefits of mathematics, especially the material taught by teachers about daily life concepts should be the learning activities carried out in class that immediately lead students to get the benefits of learning mathematics through the Contextual Teaching and Learning approach (Chen et al., 2019; Silseth & Erstad, 2018). By
developing the CTL-based HLT concept, it is expected that the students can gain knowledge through a series of construction processes and relate them to the context of the whole meaning of the topic being studied instead of memorizing only. The CTL has been proved to develop students' problem-solving abilities through critical and creative thinking processes by providing real-world problems (Clarke & Roche, 2018; Ekowati, 2015; Selvianiresa & Prabawanto, 2017). Thus, regarding the material of sequences and series, the HLT was developed by using CTL principles which consisted of HLT finding the concept of number patterns, arithmetic sequences, arithmetic series, geometric sequences, and geometric series (Widyaningsih et al., 2021; Talosa et al., 2021).

**Literature reviews**

**Hyphothetical learning trajectory (HLT)**

HLT is designed by teachers to show the learning achievement of students (Bustang et al., 2013). HLT is a learning flow of students that is predicted (hypothesized) by teachers to study certain mathematical topics. HLT consists of three components namely 1) learning objectives; 2) learning activities, media or tools used in the learning process; and 3) hypotheses or predictions to the understanding of students during the learning process, how students learn and strategies used in doing activities in the classroom (Doorman, 2018; Ivars et al., 2018; Prahmana & Kusumah, 2016). In other words, HLT is a design assessment of learning activities that begins with the level of understanding, formulation of final goals and mental activities that students bring up. At the core of HLT is the student’s action hypothesis during learning as well as the feedback for each activity raised (Handayani et al., 2019; Putra et al., 2020).

HLT is a guideline for teachers to start learning activities based on prior knowledge of the material or topic being discussed. It is said that HLT means that starting with what is already in the students towards the understanding of the concept to be achieved (Ivars et al., 2018; Simon et al., 2018). This means that prior knowledge is condition that must be considered by teachers in designing learning activities. HLT provides guidelines for teachers to set prerequisite materials, students' thinking, mathematical thinking levels and activities that can encourage students to develop their thinking to achieve learning goals (Amori, 2021; Rinartha et al., 2018).

HLT can be developed in the form of Local Instructional Theory (LIT). LIT consists of theories about the learning process on a particular topic or material and the ideas that support the learning (Gravemeijer, 2015; Larsen, 2013; Walt, 2013). LIT is a learning flow of certain mathematical materials and things that may (hypothesize) occur at every level of students’ understanding. LIT was developed through Design Research Projects which has three phases namely (a) preparing for the experiment, (b) experimenting in the classroom, and (c) conducting retrospective analyses (Gravemeijer, 2015).
Contextual teaching and learning (CTL)

The CTL is a concept of learning that can help teachers link teaching materials to a real-world situation of learners. It indicates that the CTL is a learning concept that links the lesson material with its everyday application (Jhonson, 2002; Selvianiresa & Prabawanto, 2017). Moreover, the CTL also creates a meaningful study for learners that can stimulate cognitive structure for new connections and in line with earlier experiences (Hasani, 2016; Satriani et al., 2012; Suryawati & Osman, 2017). In other words, contextual study provides the learner's learning facility for discovering learners' concrete learning experiences through active involvement in the activities of try, do, and experience themselves (O'Sullivan, 2006; Cano-Garcia et al., 2005).

Learning activities using CTL focus how an educator designs learning based on learners’ real experience through seven contextual principles of learning which are; constructivism, inquiry, questioning, modelling, reflection, learning community and contextual assessment (Ekowati, 2015; Sutupo, 2017). Based on the principles and characteristics of the CTL, it can be concluded that in learning the CTL, the process is the highly emphasized thing in gaining knowledge that is involving processes of finding, sharing, evaluating which is developed by the teacher with interesting tools and learning media (McGill et al., 1992; Schnotz & Bannert, 2003).

Problem solving skill

Problem solving skills are among the skills that learners need to have in learning math. This agrees with the National Council of Teachers of Mathematics that recommends four principles of mathematics study which are 1) Problem Solving; 2) Reasoning; 3) Communication; and 4) Connection. Problem Solving indicates of how learners tap into the ability they have at hand to solve math problems. In this case learners not only learn math but also as a means to perform and solve complex problems themselves (Joyner & Reys, 2000). The term for a mathematical problem is not only a problem of story that simply demands to remember the formula and then use it in solving problems. For more, it is also a means for teachers to hone learners' advanced thinking skills. To that end, the problems presented are problems that could involve learners' mental processes in implementing the concept of a particular series of operations or mathematical procedures. Moreover, problems are list of questions/tests with higher complexity, so learners cannot directly apply the previously mastered accounting procedure (Butterworth & Thwaites, 2013). On the matter, classifying into two types, which are: (a) problem to find; that is to find, that is to seek, determine, or acquire a particular value or object that is not known in problems and meet the conditions or conditions that correspond to the question, (b) problem to prove, which is a procedure for determining whether a statement is true or not (Polya, 1981).

Problem solving ability is a skill that can encourage students to solve mathematical problems by understanding and choosing the right strategy, then applying it to problem solving. Related to this, Robert J. Sternberg identified several processes that students go through to find solutions to mathematical
problems, namely 1) relevant selection; 2) finding procedures; and 3) identifying similarity (Butterworth & Thwaites, 2013). In other words, the process that students go through first is identifying or selecting information that is relevant to the problem provided, secondly is finding the right procedure based on existing information and third is identifying the linkage of new information with old information available to students so that they can understand the information well. The procedure/problem solving steps that can be used as a reference in learning mathematics are 1) understand the problem; 2) make a plan, 3) carry out our plan, and 4) look back at the completed solution (Polya, 1981).

Method

Research design

This study aims to develop LIT on the material of Sequences and Series. The development of LIT begins with designing learning objectives, CTL-based learning activities, and learning hypotheses that contain predictions and anticipations in the form of HLT. The HLT that has been prepared is then tested using formative evaluation. The final result of the test is the CTL-based LIT of mathematics for the sequences and series concept. The steps of developing LIT begin with a thought experiment which is thinking about the flow the students will go through, then conducting an experiment in class (instruction experiment), and reflecting the results of the experiment. If the goal has not been achieved, then it must proceed with the next thought experiment and instruction experiment on the same material. The development design LIT has three steps, namely preparing for the experiment, the design experiment, and the retrospective analysis (Gravemeijer & Cobb, 2013). In the long term period, it will be obtained LIT. This relationship is illustrated in Fig 1 below.

![Figure 1. The Gravemeijer dan Cobb Learning Model (2013)](image)

Participant

This research was taken at SMA Negeri 1 Bayang, Pesisir Selatan Regency at 30 students of class XI MIPA in the even semester of January 2021. The context of the problems contained in the learning flow was adjusted to the activities of the local community (Brown, 2016; Hilburn & Maguth, 2015).
Instructional design

The learning objectives, activities, and student thinking hypotheses which consist of predicting student answers, teacher anticipation, and the students thinking processes to get the answers are the main concept of HLT (Doorman, 2018). The objectives to be achieved in learning sequences and series are finding the concept of number patterns, arithmetic sequences and series, geometric sequences, and series, and solving contextual problems using the concepts of arithmetic and geometric sequences and series. The learning objectives are set out in the form of HLT and described in the form of learning activities based on CTL principles as well as predictions of students’ thinking flow and teacher anticipation (Bell & D’Zurilla, 2009; Voss et al., 1983).

Instrument and data analysis

Research data in the form of interviews, observation sheets, and student works were collected and summarized for analysis (Bustang et al., 2013; Fauzan et al., 2020; Gee, 2019). The analysis aims to investigate and explain how students generalize the learning activities such as the use of context, contributions, interactive, and connections to solve mathematical problems on the topic of sequences and series. Generally, the purpose of retrospective analysis is to develop Local Instructional Theory (LIT) (Hastuti & Fauzan, 2019; Larsen, 2013; Nickerson & Whitacre, 2010). At this stage, the HLT is compared to the actual student learning and the results are used to answer the problem formulation. Meanwhile, data on problem-solving abilities were obtained through giving a final test (Indriani et al., 2018; Nuraida & Amam, 2019). The test result data is used to determine the improvement of students’ problem-solving abilities (Carter et al., 1996; Abbate & Sagri, 1970).

Results

Activity I: HLT- finding the concept of triangle and quadrilateral number patterns

The first problem was to determine the number of orange arrangements sold in the market, while the second problem was about the arrangement of duck eggs in the egg basket. This activity aimed to train students to find concepts of number patterns based on certain properties or rules and also to know the types of number patterns (triangle and quadrilateral). The activity started by asking students to understand the problem of the arrangement of oranges and duck eggs. Students were asked to identify the information, then asked to find patterns in the arrangement of oranges or eggs.

The problem was solved properly by several groups as shown in Fig 2 (i). It can be seen that students were able to determine and find the pattern of triangular number arrangement based on the orange by filling in the table. Meanwhile, some students were still incorrectly finding the concept of a square number pattern as shown in Fig 2 (ii). It can be seen that students were confused in finding the relation of the number of days with eggs and it was ended by adding the numbers up even though the results are not the same. Thus, the teacher provided direction
and guidance to students if they had difficulty by reminding them about the concept of quadrilateral numbers. After finishing the group discussion, the teacher asked the group representatives to present their work in front of the class. From some of the given answers, the teacher gave stimulating questions that lead to agreement on the triangle and quadrilateral number patterns. Based on the experimental results for the discovery of the concept of triangular number patterns, it can be concluded that the contextual problems presented in learning can stimulate students in finding the concept of triangular and quadrilateral number patterns. Based on the experimental results, it can be concluded that the contextual problems presented in learning can stimulate students in finding the formal form of the triangular number pattern, namely $Un = \frac{1}{2} n (n + 1)$, and the rectangular number pattern, namely $Un = n^2$. The student settlement strategies can be seen in Fig 2 below.

![Figure 2. The Students working finding the Concept of Number Patterns](image)

**Activity II: HLT-finding the concept of arithmetic sequences**

At this meeting, students discussed finding the concept of the term on arithmetic sequences by the chili crop problem. Mostly, in solving this problem, they did not find any significant difficulties because they had also found the same thing in triangular and quadrilateral number patterns previously. Based on Figure 3 (i), it was known that the problem activities (ii) for students in finding the formula for the $n$ term of an arithmetic sequence was achieved properly. Students attempted to determine the relationship between the order of taking and the chili produced. Meanwhile, some confused students tried to follow the steps without checking the correctness of the pattern they made so that they were not able to find the concept. For this reason, the teacher reiterated how to utilize the information in the completion table by asking questions about the number patterns that have been studied previously. During group discussions, students try to find the pattern of the $n$th term by following the steps. Through problems, it can help students find the formal form of the $n$ term of an arithmetic sequence. The student settlement strategy in solving problems can be seen in Fig 3 below.
Activity III: HLT- finding the concept of arithmetic series

In this activity, students were asked to find the sum of all terms from an arithmetic series through the problem of catfish yields. In Fig 4 (i), it can be seen that students did not understand how to change the form of the Un equation to Sn, and as the result, they could not find the formula for the number of arithmetic series. This condition occurs when students are not accustomed to changing the formal form of mathematics to another. Therefore, the teacher provided more stimulating questions that lead students to understand how to change the Sn equation through the Un formula that has been studied previously. With the teacher's guidance, students were able to complete each step in the given activity as shown in Figure 4 (ii). Students have been able to utilize their past knowledge of the Un formula to help them find the Sn formula. Based on this activity, contextual problems can lead students to find the formal form of the arithmetic series concept, namely \( S_n = \frac{1}{2} n (2a + (n - 1)b) \). In solving this problem, some students had difficulty in finding the formula for the sum of the first terms of an arithmetic series as shown in the following Fig 4 below.

Activity IV: HLT-finding the concept of geometric sequence

This activity aimed to find the pattern of the \( n \) term of a geometric sequence over the problem of earning rice every year. The strategy used by students in finding
the pattern of the \( n \) term in a geometric sequence was to find the relationship between the sequence and the number formed as shown in Fig. 5 below.

![Figure 5: The Students Work in Finding the Concept of Geometric Sequence](image)

Based on Fig 5 (i) it can be seen that students found the pattern of the \( n \) term by connecting the sequence and the numbers formed. Previously, the students also learned how to find a number pattern which was not much different from finding the pattern of the \( n \) term in a geometric sequence. Meanwhile, in answer 8 (ii) students were still confused in formulating the rank of the ratio obtained. Students make a mistake in determining the correct exponent of the geometric sequence pattern. Students only try to adjust the shape of the existing pattern without paying attention to the truth of the pattern made. For this reason, the teacher guides how to determine the rank of the ratio by paying attention to the rank of the previous term so that they could find a geometric sequence pattern, namely \( U_n = ar^{n-1} \).

### Activity V: HLT - finding the concept of geometry series

The problem in this activity was the harvest of tilapia for one week. The following strategies were used by students as shown in the following Fig 6 below.

![Figure 6: The Students Work in Finding the Concept of Geometry Series](image)

In Fig 6 (i) it can be seen that students could complete the activity by finding the concept of a geometric series by following the steps. The students used their existing knowledge when they found the formula for the number of arithmetic series which is not much different. However, there was still a group of students...
who could not complete the activity as shown in Fig 6 (ii). The students seemed to know how to interpret the form of $S_n$ into $U_n$, but during the addition operation, they started to confuse and chose not to finish it. For this reason, the teacher tried to remind the concept of algebraic operations by adding up similar variables. In addition, during class discussions, the teacher also guided them to understand the concept of the number of geometric series and its use in problem-solving. For this reason, the formal form of the concept of a geometric series was obtained, namely, $S_n = \frac{a(1-r^n)}{1-r}$.

Discussion

Based on activity I, most of the students have been able to find the concept of triangular and quadrangular number patterns by following all the steps. However, there was still a group of students who incorrectly complete the activity because they did not understand the square form of numbers. Thus, the teacher provided some direction and guidance by reminding the previous material about square numbers. This way positively helps students in developing their knowledge of number patterns based on what they have learned (Fernando & Marikar, 2017; Weise et al., 2020; Zambrano et al., 2019). Furthermore, at activity II, students were asked to find the concept of the n term of an arithmetic sequence through the given problem. For the activity, some students could find the concept while still a group of students who were confused in finding the right relationship between the order of taking and the number of chilies produced. However, this condition only occurs to students with low abilities. It happens because students' initial learning abilities in understanding concepts are very diverse. In addition, students' ability to solve problems does not only depend on the type of problem given but also on the problem solver ability and conceptual knowledge that they already have (Hobri, 2021; Ince, 2018; Anif et al., 2021). For this reason, the teacher provides directions and stimulating questions so that students could find the relationship between the concepts of arithmetic sequences by reminding the previous material about number patterns.

Next, in inactivity III, students were able to find the concept of an arithmetic series through the problem of numbers of catfish. Students tried to use existing knowledge about the term of an arithmetic sequence to find the concept of numbers. However, there was a group of students that incorrectly complete the activity. It happens because students do not understand and are not accustomed to generalizing formal forms of mathematics into new concepts (Sitorus, 2016; Sumirattana et al., 2017). For this reason, the teacher provided direction by reminding the material for algebraic operations that help students find the formula for the sum of an arithmetic series.

Then, activity IV was not much different from the previous activity. In the activity, most students could find the concept of geometric sequences through contextual problems. Students followed the steps in the solution table by using the ratio of the sequence to find the formula for the n term of a geometric sequence. However, there were also unable students to complete this activity. Students were confused in finding the correct exponent form of the geometric sequence formula. For this reason, the teacher asked students to recall how to find the arithmetic sequence
concept that they have learned before. In addition, the teacher also asked students to look back at the existing rank forms and then try to find a pattern or relationship between the ratio and the order of the terms of the sequence of numbers (Prayekti et al., 2020; Risdiyanti & Prahmana, 2020).

Finally, activity V was about finding the concept of the sum of the geometric series. Mostly, the students did not experience significant difficulties in finding the formal form of the series. It happens because the previous students have also completed the same thing in activity III, namely when finding the number of arithmetic series which guide them well in finding the concept of the number of geometric series. However, there was a group of students who were unable to complete this activity because they did not understand the operations of algebraic forms, especially adding and subtracting similar variables that were difficult to succeed in finding the concept of geometric series (Rachmawati et al., 2019; Saraswati et al., 2016). Therefore, the teacher tried to recall how to combine and replace variables to find a new concept, namely the formula for the number of geometric series.

Generally, the design of HLT positively improves students' ability to find the concept of sequences and series. In addition, students can also develop problem-solving skills through giving real problems in each activity. The result shows that HLT can develop conceptual understanding and improve students' mathematical abilities (Bustang et al., 2013; Simon et al., 2018). Activities in HLT lead students to find various concepts in the material of sequences and series. Moreover, HLT is also developed based on the CTL principle where students find concepts and apply them in daily life. The students are accustomed to finding the concept of sequences and series through identifying real problems that help them to develop problem-solving skills (Barham, 2020; Ince, 2018; Olanrewaju, 2019). With CTL-based HLT, students can build their knowledge based on what they already know (Clarke & Roche, 2018; Mukwambo, 2016). It proves that the CTL learning path can develop students' problem-solving skills through critical and creative thinking processes through providing real-world problems (Selvianiresa & Prabawanto, 2017; Suciati et al., 2019).

**Conclusion**

Based on the research, it can be concluded that the CTL-based HLT is supporting students to understand the concept of sequences and series and use them in solving real-life problems. By the given problems, from the informal knowledge or experience of the students to the formal mathematics can be delivered well. Moreover, it also leads to active learning that requires students to contribute to each mathematics learning interactively and facilitate them to build their knowledge. The success of using this learning path is also answered by the final test results that show that students' problem-solving abilities can be improved properly. This means that the use of HLT has a positive impact on learning success, both the success of the process and the final success.
References


