Testing the Effectiveness of the Designing and Applying Process with Certain Circumstances in Geometry Teaching

Tran Viet Cuong
Department of Training, Thai Nguyen University of Education, Viet Nam

Dao Tam
Vinh University, Vinh City, Nghe A Province, Viet Nam

Pham Van Hieu
Department of Education and Training, Hong Bang District, Hai Phong city, Vietnam

Abstract---According to the theory of active education and Mathematics teaching in secondary schools, there are five steps designed for geometrical knowledge and four stages for the process of applying the examined circumstances. The research objective aims to evaluate the effectiveness and feasibility of the suggestion focused on the process of constructing and manipulating situations in teaching Geometry. The research method uses the case study in the specific teaching process, which was conducted with 24 teachers who have taught Mathematics at secondary schools over three months from August 2019 to October 2019; and 30 secondary school students between May 2020 and July 2020. Finally, the result shows that all teachers agree that the design and application process is appropriate and effective for their lesson. In terms of the student’s aspect, the case study reveals that the application of various steps in the process is beneficial for improving geometric deductive and spatial estimation, as well as using theoretical knowledge in the practical situation. Based on the research, this paper will clarify some solutions related to the way to design processes and utilize them in some Geometry content in secondary schools in Vietnam.

Keywords---designing process, geometry, integrated teaching, practical situations, secondary schools, teaching situations.
**Introduction**

Situational learning is a method that requires students to be active in the learning process to explore specialized concepts through the learner's knowledge and experiences (Emma Carter et al., 2021; Suryawati et al., 2010). Similarly, Piaget claimed that studying could encourage students to try new things by linking the personal understanding with the theoretical and creating their vision. In detail, when students apply mathematics to real-life situations, they may conclude that there are several hidden mathematical relations behind each problem. Moreover, to tackle those issues, they can use mathematics as a tool to perform problem-solving tasks. Therefore, students can be able to develop math skills and problem-solving abilities. In terms of a case-based learning approach, learners actively connect the study contents with the daily life context. Thus, this method could improve a better understanding of the mean for learning math (Dan et al., 2020). On the basis of students’ ability to apply knowledge to real situations, we want to test the effectiveness of the process of designing and applying geometry teaching (Gutiérrez-Rubio et al., 2020; Matiash & Mykhailenko, 2020; Cuong et al., 2020).

**Theoretical framework**

**The process of designing situations in geometry teaching according to the integrated orientation in the final grades of the secondary school**

The process of designing situations bases on the activity of the lesson investigation of lecturers through their experiential awarenesses. According to many experts, exploring the ways in which high school math teachers bring new meaning to the development of general education (Widiawati et al., 2020; Valoyes Chávez, 2019; Hau et al., 2020). Following the previous theoretical examinations, Cuong et al. (2020); Dan et al. (2020), we propose the step-by-step process below:

- **Step 1:** The teacher needs to research the content and core objectives of the lesson to be taught.
- **Step 2:** Exploring and discovering situations selected from the content of related knowledge or from other subject sources, practical situations containing information related to the upcoming lesson.
- **Step 3:** Organizing seminars in professional groups and creating polls among experienced teachers directly or through the media.
- **Step 4:** Implementing student and group surveys through the interaction with situations to detect the cognitive level of learners.
- **Step 5:** Drawing the complete situation after receiving feedback from students on cognitive activity levels: Generalization, abstraction, modelling.

**The process of applying the designed situations in the typical situations in geometry for the final grades of junior high school**

After establishing some steps in section a), the teacher can organize lessons according to the designed process. To implement the design process above, we research and propose how to apply the designed situations. The application stage needs to intentionally consider the usage of appropriate teaching methods and
theories for each content in the direction of an integrated perspective comprehension. The principal methodologies used to deliver lessons according to the integrated situations that have been conducted in this study include teaching with the viewpoint of activity theory, discovery and problem solving, cooperative teaching, teaching from the foundation of constructivist theory. Generally, these methods are popular teaching perspectives that have received much attention in theoretical and practical research on mathematics education in Vietnam.

Depending on the usage level of methods in the application process, it is essential to clarify the main activities of teachers and students in the process of interacting with integrated situations. The process of applying integrated situations in teaching Geometry will be discussed into 4 steps for the final grades of lower secondary school:

- **Step 1**: Transferring cognitive tasks to students by providing integrated situations which are appropriate to the content and objective of the Geometry topic taught.
- **Step 2**: Teacher organizes several interactive activities for students to answer the system of questions or lecturer’s orientation according to the situations (in group work or after-hours learning activities depending on the requirements and level of recognition) in order to discover concepts, propositions, make predictions, hypotheses, and models of phenomena.
- **Step 3**: Asking students to discuss and prove the judgments, testing hypotheses to solve problems in the model.
- **Step 4**: Students are free to state the conclusions about the concepts, clarify the propositions and meanings of the knowledge from the mathematical models, and set a direction for the new problems development. Finally, the teacher will confirm the above knowledge.

**Assumption**

Example 1. A triangular prism object at rest is acted upon by 3 forces with \( |\overrightarrow{OA_1}| = BC; |\overrightarrow{OB_1}| = CA; |\overrightarrow{OC_1}| = AB \), accountable by N, and the directions of the above forces are perpendicular to \( BC; CA; AB \). The mission is to prove that the object stands still.

**Scenario design is carried out according to numerous steps of the process**

- **Step 1**: The teacher determines that the core concept of teaching to solve this math exercise is applying interdisciplinary knowledge of Math - Physics to solve a practical problem.
- **Step 2**: Exploring and discovering situations chosen from relevant content or from other subject knowledge, and practical curriculum containing relation to the upcoming lesson content.

The selected situation involves the synthesis of forces to explain a certain phenomenon related to Physics.
Step 3: Organizing seminars in professional groups and polling formulation of experienced teachers directly or through the media platform.

The poll result shows that this situation is only suitable for good knowledge-level students.

Step 4: Consulting with individuals or groups of students through a small survey in a case study manner.

This exploration is mainly focusing on student’s work on solving problems with the knowledge of Physics linking to Math models.

Step 5: Complete the situation after collecting feedback from the students about the cognitive activity levels in the surveyed group solutions in the method of applying Math and Physics knowledge.

We believe that in order to make the exercise be more suitable for middle school students, the vector description needs to be expressed through ordinary language, the intensity and direction are on the same line but opposite directions.

**Teaching application**

Step 1: Students receive cognitive tasks through an integrated situation, which is appropriate to the content and objectives of the Geometry topic. This situation is revealed in figure 2. In Physics, when an object stays immovable under the action of 3 forces, the resultant force is:

\[ \overrightarrow{OA_1} + \overrightarrow{OB_1} + \overrightarrow{OC_1} = \vec{0}. \]

In terms of the figure 2 model, students will have to prove that the three points D, O, A₁ are collinear and \( OA_1 = OD \).
The task requires students to use the Geometry theory that they have learnt in the program for the last grade of junior high school to concretize the problem and find out the optimal solution.

- **Step 2**: Let students join in interactive activities with the contexts according to the questionnaires or teacher’s orientation (activities in class, in groups or out-of-school-hours learning activities depending on requirements and the awareness level of the topic that need to be taught) in order to discover formulas and rules, make assumptions, hypotheses, and models of phenomena.

- **Step 3**: Ask students to develop reasoning in order to prove the assumptions, to test hypotheses, to solve problems in the mathematical model.

To prove $OA_1 = OD$, students will have to apply geometrical knowledge about cyclic quadrilaterals, congruent triangles, properties of parallelograms. Expected answer:

$\triangle D\hat{A}_1\hat{O}$, we get $\hat{A} + \hat{O} = 180^\circ$ or $\hat{A} + \hat{C}_1O = 180^\circ$.

Since $\hat{O}\hat{C}_1\hat{D} + \hat{C}_1\hat{O} = 180^\circ$, this follows $\hat{C}_1\hat{O} = \hat{A}$.

Moreover, $\hat{C}_1\hat{D} = B_1O = AC$; $\hat{C}_1O = BA$ so $\triangle D\hat{C}_1\hat{O} = \triangle CAB \Rightarrow DO = BC$,

And $BC = OA_1$, so $DO = OA_1$.

Also, from $\triangle D\hat{C}_1\hat{O} = \triangle CAB$, we get $\hat{C}_1\hat{O} = \hat{A}$.

On the other hand, in given cyclic quadrilateral $IBKO$, we get $\hat{C}_1\hat{O} + \hat{A} = 180^\circ$.

We obtain $\hat{C}_1\hat{O} = \hat{A} = 180^\circ$, we get $\hat{D}O = \hat{A}$.

We deduce that $D$, $O$ and $A_1$ are collinear points.

- **Step 4**: Let students state the solutions to the mathematical problems, clarify the geometrical knowledge in the context which contains other science contents associated with the real world, and the meaning of the knowledge drawn from mathematical models.

**Research Methodology**

**Purpose of edagogical experiment**

The pedagogical experiment aims at evaluate the feasibility and effectiveness of the process of creating Geometry context-based problems for the final grades of secondary school in the direction of concretizing the integrated teaching approach, the process of applying the contexts created, typical mathematical contexts in Geometry in the last grades of secondary school, and the results of applying the created contexts in teaching Geometry for the last grades of secondary school in an integrative orientation.

**Method of pedagogical experiment**

We conduct experiments: select lessons, conduct experiments and evaluate in the form of case studies for one or several groups of Math teachers and students in the last grade of secondary school. In the case study, we focus on the forms of group research, discussion and debate, the experimenter will observe the behavior of students and teachers throughout activities corresponding to the lesson that is experimentally transferred to the selected groups. The experiment
conductor gives questionnaires and instructions, observes, listens, and records the activities of each individual selected, and records discussions and debates of teachers and students to find out about beliefs, attitudes, thinking activities, ideas sharing in groups and the solutions of those groups. The above factors will be the basis for qualitative assessment of understanding, teaching and learning ability from the teaching point of view of teachers and students (Widiasri et al., 2019; Tuarez et al., 2019).

**Experimental content**

Experimental content is assigned to groups of teachers and students.

- **For teachers**
  We conduct experiments in the direction of concretizing the process of creating contextual teaching combined with teaching concepts, theorems and rules. To be specific, the experiments are as follows:
  - + Experiment 1: The process of creating lesson plan to teach the concepts: "Two shapes being axisymmetric and a symmetrical shape".
  - + Experiment 2: The process of applying teaching situations to teach the concepts: "Two shapes being axisymmetric and a symmetrical shape".

- **For students**
  When experimenting teachers, in the part of applying the process, there are a number of activities that students need to conduct to make assumptions, hypotheses, and do activities to test assumption and hypotheses. These activities are conducted regarding the instructions and questionnaires of teachers. For the above reasons, in the experiment for students, we only pay attention to students' activities which are interactive activities with contexts to explore and discover knowledge. The content of the experiment for students is as follows:
  - + Experiment 1: Explaining practical contexts through mathematical modeling activities.
  - + Experiment 2: Using esoteric integration to form the theorem about cyclic quadrilaterals.

**3.4. Experimental tools**

Giving contexts and questionnaires.

- **For teachers**
  - Experiment 1: We give contexts and questionnaires
    Let groups of teachers observe the following figures: a football field with the midfield line going through the center spot; a window frame; a circular a card with diameter AB; line segment AB with midpoint O in figures 3, 4, 5 and 6, relatively.
Study the teaching objectives, choose which figure to use as a context to teach the concepts of "Two shapes being axisymmetric and a symmetrical shape".

Conduct a discussion on how to choose appropriate context to teach the above concepts.

Conduct surveys among students to make a choice of the contexts that are appropriate with students' characteristics of cognitive.

Confirm to choose the context that the teacher use to teach the lesson “Two shapes being axisymmetric and a symmetrical shape”.

From the a priori analysis, we expect the experimental groups to choose the options of figure 3, figure 5, and figure 6.
Experiment 2: We give contexts; questionnaires and instructions

- The experimenter transfers information to groups of teachers and analyzes the contexts they have chosen.
- Teachers let students interact with the contexts by giving questionnaires and instruct students to interact with the contexts to point out: two shapes being axisymmetrical and a symmetrical shape, with the expectation that students have to do the modeling: Describe the average line $EF$ - the line containing the midfield line of the football field $EF$ is the axis of symmetry of the two rectangles $EFDA$ and $EFCB$.

![Figure 7. Rectangles EFDA and EFCB](image)

![Figure 8. Circular card with diameter AB](image)

![Figure 9. Line segment AB with midpoint O](image)

- The line $AB$ is the axis of symmetry of the two semicircles $(AMB)$ and $(ANB)$.
- The perpendicular bisector $(d)$ of the line segment $AB$ is the axis of symmetry of the two figures - line segments $OA$ and $OB$.

Require the experimental teacher to ask questions to students: How can we prove rectangle $ABCD$; circle with center $O$; line segment $AB$ are shapes with axes of symmetry? Regarding the consideration of the above cases, the conductor requires experimental teacher groups to ask questions so that students can state the general definition of a shape with a symmetry axis (Danchikov et al., 2021; Lian, 2021).

For students
- Experiment 1: Let students approach the contexts
Description of an equipment for lifting and lowering objects which is shown in Figure 10:

![Equipment for lifting and lowering objects](image)

**Figure 10. Equipment for lifting and lowering objects**

- The basic characteristics of the lifting device include: two iron bars of equal length are connected by an axis going through their midpoints; two iron bars move around the axis; the endpoints of two iron bars are always the 4 vertices of a rectangle A, B, C, D. When raised, the length increases and the width decreases. Can you explain this phenomenon using mathematical models?

- Experiment 2: Consider the context
Consider the context: Given square ABCD; rectangle ABCD, isosceles trapezoid ABCD with bases AD and BC.

![Square ABCD](image)

**Figure 11. Square ABCD**

![Rectangle ABCD](image)

**Figure 12. Rectangle ABCD**

![Isosceles trapezoid ABCD](image)

**Figure 13. Isosceles trapezoid ABCD**
• Reason in different ways to prove that square, rectangle, and isosceles trapezoid are inscribed in a circle.
• State the common characteristics of the three figures that could be inscribed in the circle.
• Make a general statement about the condition for a quadrilateral to be inscribed in a circle.

**Survey form**

We conducted a survey among 24 teachers during the period from August 2019 to October 2019 at three schools: Hong Bang Secondary School; Nguyen Trai Secondary School and Nguyen Binh Khiem Secondary School. We conducted a survey among 30 students during the period May 2020 and July 2020 at three schools: Tran Phu Secondary School; Nguyen Ba Ngoc Secondary School and Nhan Hoa Secondary School (Widana et al., 2020).

**Survey operation**

We transferred contexts, questionnaires for groups of teachers and students to study and discuss to find solutions. The teacher was in charge of experimentally observing the activities of the students. Teachers recorded audio and video of some interactions of students with situations; interact with students, share and discuss ideas to solve problems. Teachers recorded audio and video of conclusions and knowledge discovery of groups (Tiberghien, 1994; Hofer et al., 2021).

**Research Results**

**Results of teacher experiment**

Experimental results are mainly evaluated qualitatively and quantitatively through the evaluation of answers to experimental situations given and obtained results.

• Experiment 1: After observing and discussing with three groups of seminar teachers, we obtained the following analysis results: Teachers knowing the planning process in 5 steps:
  • There are 22/24≈92% of teachers who have caught the idea of the process of creating contextual-based problems in teaching Geometry in the final grades of secondary school in the direction of concretizing integrated teaching.
  • There are 2/24≈8% of teachers who still have difficulty in step 2, which is to explore and discover contexts selected from related knowledge content or from other subjects’ knowledge content, and practical contexts containing relevant knowledge associated to the lesson to be taught.
  • The cause is that this group of teachers is still inexperienced in teaching with only 2 to 4 years of working experience, so they do not have enough time and experience to accumulate knowledge, as well as the relate mathematical knowledge to other science subjects and exploit potential...
real-world contexts containing mathematical knowledge related to experimental situations.

- In the selection of contexts related to the situation: \( \frac{24}{24} = 100\% \) of teachers selected the contexts in figure 3; figure 5; figure 6. This is also something we anticipated before.
- In Case 1, we discovered that \( \frac{16}{24} \approx 67\% \) of teachers had difficulty indicating that each point \( P \) belonging to a semicircle \((AMB)\) has an image of \( P' \) belonging to a semicircle \((ANB)\) and opposite, each point \( Q \) belonging to a semicircle \((ANB)\) has an image of a semicircle \((AMB)\). This is also mentioned in the a priori section because almost all teachers here are unconcerned about the proposition of central congruence.

- Experiment 2: During the discussion and notes of teacher’ groups involved in experiment 2, we realized that \( \frac{24}{24} = 100\% \) of teachers had begun to understand the process of applying the designed situations in teaching and the standard situations in Geometry in the final grades of Secondary school. Teachers discuss and consider the use of suitable teaching methods and theories for each content to be taught in an informed approach. The main methods used to implement teaching according to the integrated situations designed in this study include teaching from the perspective of activity theory, teaching finding and solving problems, collaborative teaching, teaching from the perspective of constructivist theory, and giving the level in the use of methods in the application process to clarify the main activities of teachers and students in the process of interacting with integrated situations. There are \( \frac{23}{24} \approx 96\% \) of teachers who already have a plan to transfer cognitive tasks to students by providing integrated situations that become relevant to the subject and goals of Geometry Case 2. There are still \( \frac{1}{24} \approx 4\% \) of teachers who are having difficulty transferring tasks to students, which is understandable given their lack of teaching experience, they have specialist skills and a thorough grasp of the process, but lack teaching experience; this will be overcome by teachers in the subsequent instruction. \( \frac{24}{24} = 100\% \) of teachers have been organized for students to interact with situations according to the questions system or teachers orientation in groups, and students have found notions, propositions, given predictions, hypothesis testing, models of phenomena as mentioned previously. Students argue to verify decisions, test hypotheses, resolve issues in the model of situations, state conclusions about notions, and make clear propositions and knowledge meanings are derived from mathematical models, moreover, they also provide the orientations for the development of fresh problems.

**Students’ experimental outcomes**

- Experiment 1:
  In situation 1, \( \frac{20}{30} \approx 67\% \) of all the students in the three groups conducted modelling the situation when observing the movement of the lifting equipment. The students described and modeled the situation actively: Two
iron bars of equal length are connected by an axis through their midpoints, the ends of the iron bars are always four vertices of a rectangle: \( A, B, C, \) and \( D \). It can be used to model the phenomenon of lifting and lowering equipment in figure 4.19 and figure 4.20 as two iron bars \( AC \) and \( BD \) are represented by two diagonals of \( ABCD \) rectangle, shown in Figure 4.26. The nature of lifting the object is that as the forces acting on the width \( BC \) decreases, so does the length \( AB \) ascends \( AB \leq AC \). When the two diagonals start moving, the shape of the rectangle changes; in fact, the length and width of the rectangular shape transition according to the rule. The Pythagorean theorem is used to describe this rule: \( AC^2 = AB^2 + BC^2 \) (1). Because \( AC \) has a length defined by the length of the iron bar, equation (1) implies that the larger \( BC \) the smaller \( AB \), hence the corresponding device to lower the object. In contrast, the smaller \( BC \) the larger \( AB \), corresponding to the device lifting the object.

There are 10/30 ≈ 33% of students who struggle with modeling because it is difficult to understand the idealization operation. The iron bar represented by the segment line is the students who weaken in applied thinking to solve feasible phenomena described in Mathematical languages. While they had direct connections to the teacher’s instructions, they actively described and modeled the situation (Burgess & Spencer, 2000; Rodriguez et al., 2018; Aven, 2017; Yager et al., 2013).

- **Experiment 2:**
  In context 2: Given \( ABCD \) square, \( ABCD \) rectangle, and \( ABCD \) trapezoid triangle with bases \( AD \) and \( BC \).
During the process of observing and interviewing students, we calculated that 30/30 = 100% of pupils mentioned the similar elements of three regular shapes that could incircle the circle and explain the incircle squares and rectangles at least equal to each of two methods using the knowledge that was learned (Conrad, 1990; Gilmore, 1972).

- Method 1: Students used the definition of a circle which was learned in sixth grade to demonstrate the specific inscribed circles as follows: Squares and rectangles have equal diagonals and intersect at the $O$ midpoint of each line, as shown in figure 15, figure 16 infer that $OA = OB = OC = OD$. Hence, $O$ is the center of the square and rectangle circumscribed circle.
- Method 2: The vertices $B$ and $D$ of the square and rectangle both look at the two ends in the diagonal $AC$ under a right angle. Hence, the square and rectangle inscribed in a circle have diameters that correspond to diagonals $AC$.

There are 6/30=20% having difficulty in mentioning that an isosceles trapezoid can inscribe the circle while explaining that the perpendicular bisectors of the sides $AD$ and $BC$ of $ABCD$ trapezoid coincide (see figure 17). Let $I$ be the intersection point of the perpendicular bisector $d'$ of $AB$ side and perpendicular bisector $d$ of $AD$. Likewise, $d$ is also perpendicular to $BC$ at $J$ because $BC /\parallel AD$. We can verify that triangle $\triangle BIC$ is an isosceles triangle at $I$. Again, $IJ$ is the altitude and the perpendicular bisector. So, the two perpendicular bisectors of $AD$ and $CB$ coincide. There are 30/30 = 100% of students who have stated that the sums of opposite angles of quadrilaterals are equal to $180^\circ$ and have stated the quadrilateral definition inscribed in a circle (Vayre et al., 2012; Kutin et al., 2018).

Discovering

The teaching method through the process of designing and applying to situations for teaching Geometry contributes to improving effectiveness, confidence, and interest in mathematics in students, as well as supporting knowledge integration.
of math and other sciences. The teaching method through the process of designing and applying to situations for teaching Geometry in secondary schools in Vietnam has attempted to increase students’ ability to solve the situations in the real context and bring a lot of practical values to students.

Middle school range is regarded as a critical stage in the growth of student values and inclinations. Therefore, there is a need for a better understanding of the improvement of the integrated education program as well as an investigation into students’ awareness of integrated learning. This article refers to the growth of the ability to connect real-life situations with mathematical applications. The findings of this study demonstrate the importance of developing the values and benefits of integrated education with students, and improving an integrated education program based on a variety of life-related topics that surround us.

Conclusion

The research findings are consistent with the results of (Emma Carter et al., 2021; Suryawati et al., 2010; Cuong et al., 2020; Dan et al., 2020). Hence, the research team recommends to the Ministry of Education and Training that Secondary schools in Vietnam should encourage teachers to actively innovate teaching methods. In there, it is to base teaching on situations designed by teachers using the process proposed in this study. We believe that by process designing and applying to real-life situations in Vietnamese Middle schools, students’ learning outcomes in terms of how they understand knowledge and apply solutions to address real-life issues will improve.

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